Few-Body Systems Suppl. 8, 1–5 (2008)

Few-Body Systems

An Analysis of the Use of the Cluster Separability in Scattering Theory

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nucl-th/9511014

Abstract. In the framework of the Time Dependent Scattering Theory we discuss three forms of Cluster Separability as well as the conditions for the representation of the scattering system dynamics implied by their respective use.

For a broad class of theories applied to the studies of the scattering processes of particles and their bound states, and in particular for the Quantum Field Theory (QFT), one needs to include the procedure which regulates a segmentation and clusterisation of the scattering process. Implications of the implementation of such a procedure as well as its relationship to the general formalism deserve a careful analysis. In this paper we would like to point out the relationship existing between Cluster Separability (CS), Adiabatic Cluster Separability (ACS) and Disturbative Adiabatic Cluster Separability (DACS). The last one being introduced by us in the previous publication [1]. These three competing procedures are stated in the form of a principle for scattering theory as conditions for the representation of the scattering system dynamics.

The common feature of all of them is that they imply distinction between central interaction channel of the system and two channels corresponding to the initial and final subsystems respectively. Channels could be described by corresponding Hamiltonians and states in their corresponding Hilbert spaces.

What makes a difference between CS, ACS and DACS is the degree and a way of describing the "phase transitions" between the corresponding segments of the scattering process.

We will consider the scattering process $\alpha \to \beta$ in the framework of Time dependent propagator theory. Initial and final channels of the process are defined

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with the sets of the process constituents - particles and their bound states: $\{I_1, ..., I_k\} = \alpha$; $\{F_1, ..., F_l\} = \beta$. A corresponding Hilbert space state is denoted by $\mid \alpha >$. The Bethe-Salpeter amplitudes χ_α, $\bar{\chi}_\alpha$ for channel α and process propagator $G_{\alpha\beta}$ are:

$$\chi_\alpha(n) = < 0 \,|\, T\big(\psi(x_1)...\psi(x_n)\big) \,|\, \alpha >; \;\; \bar{\chi}_\alpha(n) = < \alpha \,|\, T\big(\psi(x_1)...\psi(x_n)\big) \,|\, 0 >, \quad (1)$$

$$G_{\alpha\beta}(n;m) = < 0 \mid T\Bigg(T\big(\psi(x_1)...\psi(x_n)\big) \; T\big(\psi(x'_1)...\psi(x'_m)\big) \Bigg) \mid 0 > . \qquad (2)$$

In order to reduce general forms (1), (2) to the working expressions, one needs to perform as well the following steps:
1. Functional derivation of the $G_{\alpha\beta}(n;m)$. It allows an expression for the process propagator $G_{\alpha\beta}(n;m)$ through the cluster propagators G_i, G_f and establishes the initial frame for the cluster decomposition of the process.
2. Corresponding Gell-Mann, Low limiting procedure [2]. It assures the QFT consistent contact with a states of the system for remote times.
3. Application of the one of the above mentioned procedures (CS, ACS, or DACS). It results in the segmentation of the scattering process and also defines the final form of the cluster decomposition.

The exposed steps which are defined for the general propagator imply the decomposition of the corresponding S-matrix as well as the corresponding Feyman developement.

We will focus our attention on the third procedure which equips the system with the corresponding form of the cluster separability: unspecified, adiabatic or disturbative adiabatic.

Cluster Separability is the characteristic that a system of particles when broken into spatially remote subsystems should be such that the dynamics of each subsystem are independent of each other [3]-[4]. (This property is also denoted as Separability of the Interaction or Cluster Property.)

The outcome of the CS is a flexible frame for the representation of the systems dynamics. In addition it does not introduces any demands for the "contact processes". A freedom for the interpretation of that part of a scattering formalism still remains.

The definite demands for the segmentation of the scattering process are established with the implementation of ACS.

Adiabatic Cluster Separability: "No error is introduced into the treatment of the physically realizable scattering process by a formulation of the theory in which the interactions among the particles of interest are 'turned off' at remote times-provided, of course, that the turning-off procedure is sufficiently gradual that it does not, of itself, create disturbances" [5]. (The standard name is Adiabatic Hypothesis.)

The configurational content of ACS is a regionalization of the process in a space-time. The initial (or incoming) region of propagation $\mathcal{R}_i$, the interaction region $\mathcal{R}_k$ and the final (or outgoing) region of propagation $\mathcal{R}_f$ are distinguished. The consequent factorisation of the process representatives (propaga-

tor and S-matrix) is expressed in relations:

$$G_{\alpha\beta} = G_\beta R_{\alpha\beta} G_\alpha = \prod_{f=1}^{l} G_f R_{\alpha\beta} \prod_{i=1}^{k} G_i; \tag{3}$$

$$S_{\beta\alpha} = \bar{\chi}_\beta^{out} R_{\alpha\beta} \chi_\alpha^{in} = \prod_{f=1}^{l} \bar{\chi}_{\beta f} R_{\alpha\beta} \prod_{i=1}^{k} \chi_{\alpha i}. \tag{4}$$

In Eqs. (3)-(4) G_α, G_β are channel propagators, χ_α^{in}, $\bar{\chi}_\beta^{out}$ are the remote times amplitudes, and $R_{\alpha\beta}$ is a truncated propagator. G_i, G_f, $(\chi_{\alpha i}, \bar{\chi}_{\beta f})$ are propagators, (amplitudes) of the scattering process constituents. A segmentation of the scattering process is expressed on the left-hand side of the above expressions. A cluster decomposition is seen on their right-hand side.

A strict application of the ACS implies the sharp distinction of three propagation regions. Since the propagation is adiabatic, for the whole regions $\mathcal{R}_i$ and $\mathcal{R}_f$ the observable characteristics of the system are the same as they were or will be at the remote times $\mp\infty$.

We argue that such a feature introduces serious limitations for the consistent representation of the scattering processes involving bound states. Particularly critical would be the description of the intermediate objects: transition operators or segments of the convolutional expresions for the form factors, for instance. Additional problems are related to the interpretation of the contact processes which are reduced to a cut-off.

In this way one can recognise a limitations for the incorporation of the ACS into the formalisms which include deconfinement and hadronization. In particular, it concerns the formalisms which contain on the basic level different or hybrid levels of compositeness.

We extract three additional groups of motivs for the reformulation of ACS with a less rigid principle.

I) Motivs of the conceptual nature. The intermediate phases have been already tacitly introduced (by using, for example, off mass shell contributions or even intermediate clusterisations of the channels) without explication whether it is consistent or not with the general procedure.

II) In applications of the standard formulation one often finds the incompatibility of the interaction effects originating from scattering amplitudes and from truncated propagators.

III) The idea was to find consistent and in the same time flexible formulation which could be applied to many concrete scattering situations.

The rigidity of the ACS and its limitations in encompassing the intermediate segments led us to the conclusion that one should formulate a different principle for the segmentation of the scattering processes.

We retained the supposition that interaction between constituents of scattering processes could be switched off at the remote times, or precisely for the initial and final space-time regions of propagation. But in addition we included a new one by which the mechanism of inclusion/exclusion is such that

it creates the disturbances for initial/final channels in the vicinity of the interaction region. We supposed also that disturbation effects could be represented by segments obtained in a factorization of the scattering picture i.e. that the transition from initial/final regions of propagation to the region of "pure" interaction, generally, is not direct but it goes over the intermediate disturbed phases.

Disturbative Adiabatic Cluster Separability: In the framework of the Quantum Field Theory, propagation process can be, generally, represented by a few-step mechanism. Finite intermediate regions of disturbance which can be represented by a superposition of the physically realizable configurations for the corresponding channels are compatibly connected with the central, interaction region. Interaction among the constituents of the scattering process is 'turned-off' for the initial and final propagation regions, so that initial and final configurations are corresponding to them [1]. (The other name is Disturbative Adiabatic Hypothesis.)

By incorporation of the DACS in the scattering theory space-time content becomes correspondingly reacher. Two additional domains of scattering process appear. Disturbed channels of the initial and final configurations correspond to the initial and final disturbation regions $\mathcal{R}_{id}$ i.e. $\mathcal{R}_{fd}$. So one can visualize the sequence of the five regions: $\mathcal{R}_i, \mathcal{R}_{id}, \mathcal{R}_k, \mathcal{R}_{fd}$ and $\mathcal{R}_f$ separated by the four space-like surfaces. DACS causes the corresponding factorization of the representatives of the scattering theory.

In our formulation the propagator for the process $\alpha \to \beta$ is written

$$G_{\alpha\beta} = G_\beta \bar{K}^d_\beta R_{\alpha\beta} K^d_\alpha G_\alpha = G_\beta M_{\alpha\beta} G_\alpha. \tag{5}$$

Here $\bar{K}^d_\beta$ and K^d_α describe the effect of disturbation introduced. On the right hand side of Eq. (5) the expanded interaction term, $M_{\alpha\beta}$, $M_{\alpha\beta} = \bar{K}^d_\beta R_{\alpha\beta} K^d_\alpha$, is defined.

The S-matrix gets the form

$$S_{\beta\alpha} = \bar{\chi}^{out}_\beta \bar{K}^d_\beta R_{\alpha\beta} K^d_\alpha \chi^{in}_\alpha = \bar{\chi}^{out}_\beta M_{\alpha\beta} \chi^{in}_\alpha. \tag{6}$$

The new objects, "disturbation amplitudes" read: $\chi^d_\alpha \equiv K^d_\alpha \chi^{in}_\alpha$; $\bar{\chi}^d_\beta \equiv \bar{\chi}^{out}_\beta \bar{K}^d_\beta$.

We derive mathematical basis for the disturbative adibatic (DA) formalism on the level of the QM S-matrix theory by using the method of the wave (or Møller) operators.

The contact with experimental situation is model dependent. The interaction is given with the form of the expanded interaction term $M_{\alpha\beta}$. In addition it is necessary to define the disturbation effects. The remote time amplitudes χ^{in}_α and $\bar{\chi}^{out}_\beta$ are determined phenomenologically.

The general scheme of the DACS could be projected on the various scattering formalisms. The disturbation input is determined with appropriate superposition of the physically realizable configurations for the corresponding channels. The concrete examples of possible applications are the off mass shell effects in nuclear reactions, the formation of the new particles states, the transition amplitudes at the quark level and the appropriate intervention in the Fock

expansion. The interaction content could be described by the corresponding Lagrangian.

The general formalism contains also two procedures which are independent and mutually compatible.

One of them consist in trying to find "natural" representation of disturbation effects in the form: $K^d = G^d V; \quad \bar{K}^d = \bar{V} G^d$.

Propagators G^d are related to the finite domains. They have the same physical meaning as propagational segments of a truncated propagator $R_{\alpha\beta}$ which appear in the course of the functional derivation. V, $\bar{V}$ are the vertex type operators defined by the algoritm. Expanded interaction term $M_{\alpha\beta}$ becomes:

$$M_{\alpha\beta} = \bar{V} G^d R_{\alpha\beta} G^d V. \tag{7}$$

One can recognize that the form of the expanded interaction term (7) corresponds to the convolution structure of the form factor. As an example of this structure one can use the nucleon form factor described in elastic electron-nucleon scattering within the QCD hard scattering scheme.

The second procedure consists in the series expansion of $M_{\alpha\beta}$ and reads

$$M_{\alpha\beta} = \big(\bar{K}_\beta^{d(0)} + \bar{K}_\beta^{d(1)} + ...\big)\big(R_{\alpha\beta}^{(0)} + R_{\alpha\beta}^{(1)} + ...\big)\big(K_\alpha^{d(0)} + K_\alpha^{d(1)} + ...\big). \tag{8}$$

This expansion is not perturbative series but it reflects the content of Bethe-Salpeter equation with the precise meaning of spatio-temporal composition of interaction effects.

One needs an additional analysis of terms in (8) in order to achieve a compatibility of the interaction contributions independently of their origin.

The CS, ACS and DACS could be considered as three different formulations of the general Principle of the segmentation of scattering process. This principle one could include among the first principles of the Quantum Field Theory.

The configurational features of this analysis are the part of more general approach which also includes the studies of covariant propagation and its correlation to the forms of dynamics. Some segments of that study has been already reffered [6], [1].

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